Calculation of the Universal Gravitational Constant *G*

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In his hypothesis of 1974 concerning large numbers, Dirac noticed that certain numbers seem to reappear in many calculations [1]. *These large numbers are actually proportionality ratios that enable us to transpose certain ratios (forces, energy, dimensions, etc.) from the microscopic world into the macroscopic world and vice and versa* [2].

Nature is simple and often repeats itself as much in the infinitely large scale as in the infinitely small one. An electron rotates on itself and around the nucleus (proton) in an atom. The universe rotates on itself. Each of these rotations allows tiny "tornadoes" in space-time to increase the mass in a quantum manner by repeated relativistic effects. Even if we do not know all the processes of rotation and the quantum leaps existing between the infinitely small and the infinitely large, there seems to be 57 in total (this number is necessarily an integer). It follows that N, which is equal to the largest number described in the Dirac hypothesis, equals $6.3034195351 \pm 0.00000012 \times 10^{121}$. *Having a better knowledge of this number enables us to calculate precisely the universal gravitational constant and to evaluate it to be* $G \approx 6.6732309 \pm 0.0000003 \times 10^{-11} m^3/(kg \cdot s^2)$. *This value is consistent with the value of the CODATA 2010* [8] *which is* $G \approx 6.67384 \pm 0.00080 \times 10^{-11} m^3/(kg \cdot s^2)$.

The fact of knowing precisely the constant G enables us to reevaluate many parameters of the universe like the Hubble constant H_0, *the average temperature of the cosmic microwave background of the universe T, the apparent mass of the universe* m_u, *the apparent radius of curvature of the material universe* r_u, *the apparent radius of curvature of the luminous universe* R_u, *the acceleration of light* a_L *and the Pioneer acceleration* a_p.

KEY WORDS: Universal gravitation, *G*, Dirac, the theory of large numbers, Hubble, H_0

1. INTRODUCTION

According to a document that we have shown previously, Dirac's hypothesis concerning large numbers seems to have real foundations. According to this hypothesis, some numbers (all obtained from a single number that we have baptized *N*) show up frequently when we look for scale factors at the level of the universe. For this reason, if it were possible to know the exact value of *N* by a method other than the ones known, we could easily calculate the value of the universal gravitational constant *G* as well as all the parameters that are derived from *G*.

In this paper, we will focus on finding an independent method of calculation that can lead to the number *N*. To do so, we will use what we consider to be the best estimate of the Hubble constant, that is, a value that we ourselves have calculated in previous works. Then, knowing a good approximation of *N*, we will establish a strategy to calculate its exact value and thus obtain the value of the universal gravitational constant *G* which corresponds to it. We will then try to demonstrate that our equation relating *N* to the constant of fine structure α is true. Using the precise value of *G* that we have calculated, we will reevaluate several parameters of the universe like the Hubble constant H_0, the average temperature of the cosmic microwave background of the universe *T*, the apparent mass of the universe m_u, the apparent radius of curvature of the material universe r_u, the apparent radius of curvature of the luminous universe R_u, the acceleration of light a_L and the Pioneer acceleration a_p.

2. DEVELOPMENT

2.1. Theoretical Hubble Constant from Previous Works

In previous works [2], we obtained an equation that allowed us to calculate the Hubble constant H_0 [12] with a precision which depended mainly on the universal gravitational constant *G*:

$$H_0 = \frac{G \cdot m_e \cdot \beta^{\frac{3}{2}}}{c \cdot \alpha \cdot r_e^2} \approx 72.10 \pm 0.009 \text{ km/(s·MParsec)} \tag{1}$$

The following values come from the CODATA 2010 [8]:

- Planck constant $h \approx 6.62606957\times10^{-34}$ J·s
- Actual speed of light in vacuum $c \approx 2.99792458$ m/s
- Universal gravitational constant $G \approx 6.67384\times10^{-11}$ m^3/(kg·s^2)
- Fine structure constant $\alpha \approx 7.2973525698\times10^{-3}$
- Boltzmann constant $k_B \approx 1.3806488\times10^{-23}$ J/°K
- Classical radius of the electron $r_e \approx 2.8179403267\times10^{-15}$ m

The constant β, for its part, represents the ratio between the speed of expansion of the material universe and the speed of light [3]. The value of β is an irrational number.

$$\beta = 3 - \sqrt{5} \approx 0.764 \tag{2}$$

Several research teams around the world have developed their own way of measuring the Hubble constant and get results that they hope to be as accurate as possible. With hindsight, we also find that some results are probably presented with margins of error which do not overlap. Not knowing all the details that led to these results, it becomes difficult to give more credit to the one or the other measurement method.

Our method to obtain H_0 does not come from direct measurements [2]. It supposes, among other things, that there is a theoretical link between this parameter and the universal gravitational constant *G*. If the theoretical link that we found is good, then the margin of error is almost entirely based on the constant *G* since the latter is much larger than the other fundamental constants used.

Since the assumptions of this paper are based on evidence that there are certain numbers depending on H_0, the accuracy of this parameter seems crucial. If all the assumptions we have made in the past are true, it is logical that we continue to use these calculation results… until we are faced with a phenomenon which invalidates what we have found.

Being nonetheless concerned about showing values that are supported by independent research, let's note that the value of H_0 obtained in **(1)** is consistent with that measured by the Xiaofeng Wang team [6] which obtained $H_0 \approx 72.1 \pm 0.9$ km/(s·MParsec).

2.2. Coincidence Noticed by Dirac

In order to possibly get a second equation for the value of the Hubble constant, let's analyze a coincidence discovered by Dirac in 1974 [1].

Using the value of H_0 presented in **(1)**, the apparent mass of the universe that we can observe is equal to [4,5]:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.73 \times 10^{53}\,\text{kg} \qquad \textbf{(3)}$$

Assuming that the luminous universe [3] is currently expanding at the speed of light *c* in vacuum [7], the apparent radius of curvature of the luminous universe is:

$$R_u = \frac{c}{H_0} \approx 1.28 \times 10^{26}\,\text{m} \qquad \textbf{(4)}$$

The largest unit of distance existing in the universe thus becomes the circumference of the universe itself. Therefore, the smallest mass that can exist is the one that we can associate to a photon of wavelength equal to $2\cdot\pi\cdot R_u$:

$$m_{ph} = \frac{h}{2\cdot\pi\cdot R_u} = \frac{h\cdot H_0}{2\cdot\pi} \approx 2.72\times10^{-69}\,\text{kg} \quad \textbf{(5)}$$

Let's consider the number N as being the maximum number of photons of wavelength $2\cdot\pi\cdot R_u$ that can exist in the universe. If the universe has an apparent mass m_u and the photon a wavelength $2\cdot\pi\cdot R_u$, the mass m_{ph}, the number N is equal to:

$$N = \frac{m_u}{m_{ph}} = \frac{2\cdot\pi\cdot c^5}{G\cdot h\cdot H_0^2} \quad \textbf{(6)}$$

Let's try to define N by replacing H_0 by the equation **(1)**:

$$N = \frac{2\cdot\pi\cdot c^7\cdot\alpha^2\cdot r_e^2}{G^3\cdot h\cdot m_e^2\cdot\beta^3} \approx 6.30169\times10^{121} \quad \textbf{(7)}$$

N is a scale factor. It has no units. Its precision now depends mainly on the precision of G which is, overall, relatively poor compared with the high degree of accuracy of the other parameters such as the speed of light c, the fine structure constant α, the classical radius of the electron r_e, the mass of the electron m_e, etc. The value of G included in the CODATA 2010 [8] is $G \approx 6.67384\pm0.00080\times10^{-11}\text{m}^3/(\text{kg}\cdot\text{s}^2)$.

The fine structure constant α is of same type as the constant N, that is to say, it is also a dimensionless scale factor. The fine structure constant represents the ratio between the classical radius of the electron and the Compton radius of the electron.

Nature is simple and often repeats itself. Electrons rotate on themselves. In atoms, electrons revolve around the nucleus. The universe rotates. These are just a few examples. There are probably others that we do not know. Some of these rotations can be expressed in terms of the fine structure constant α. There are also quantum leaps that can be described in terms of the fine structure constant.

Even if the number of leaps or rotations remains unknown, the phenomenon is quantum and there cannot be any half measures. The number of times that these phenomena are used by nature is necessarily an integer. The Lorentz factor is thus applied *n* times (*n* being an integer) to the mass associated with the photon which has a wavelength $2\cdot\pi\cdot R_u$ to give the apparent mass of the universe that we know. If, in the present case, the fine structure constant α is associated with the Lorentz factor, our hypothesis can be translated by the following equation:

$$m_u = \frac{m_{ph}}{\alpha^n} \qquad \textbf{(8)}$$

So, according to equations **(6)** and **(8)**, we have:

$$N = \frac{m_u}{m_{ph}} = \left(\frac{1}{\alpha}\right)^n \qquad \textbf{(9)}$$

Let's isolate the value of *n*:

$$n = Integer\left[\frac{Log(N)}{Log\left(\frac{1}{\alpha}\right)}\right] \qquad \textbf{(10)}$$

$$n = Integer\left[\frac{Log\left(6.30169\times10^{121}\right)}{Log\left(\frac{1}{7.2973525698\times10^{-3}}\right)}\right] = Integer[56.99994] = 57 \qquad \textbf{(11)}$$

This result depends indirectly on the precision of *G* which has been used to calculate *N*. However, it is precise enough to make us sure that the integer number we are looking for is exactly 57. Consequently:

$$\left(\frac{1}{\alpha}\right)^{57} = N \qquad \textbf{(12)}$$

2.3. Calculation of the Universal Gravitational Constant *G*

It is possible, with the help of the Planck constants, to demonstrate that:

$$G = \frac{c^2\cdot l_p}{m_p} \qquad \textbf{(13)}$$

Here, l_p and m_p represent respectively the Planck length and the Planck mass:

$$l_p = \sqrt{\frac{h\cdot G}{2\cdot\pi\cdot c^3}} \quad \text{and} \quad m_p = \sqrt{\frac{h\cdot c}{2\cdot\pi\cdot G}} \qquad \textbf{(14)}$$

The problem, if we seek the value of *G*, is that the Planck units are themselves defined from *G*. We are going around in circles. It would be interesting to find a ratio of numbers having the same units that would be totally independent of *G*.

Let's start from the following equation (described in our works [2]) which is based on Dirac's large number hypothesis [1]:

$$\frac{E_k}{E_g} = \frac{q_e^2 \cdot \alpha}{4 \cdot \pi \cdot \varepsilon_0 \cdot G \cdot m_e^2} = \beta \cdot N^{1/3} \tag{15}$$

Let's try to rewrite this equation as a function of the classical radius of the electron r_e. The mass of the electron m_e is almost entirely due to its electrostatic energy. This implies that:

$$m_e \cdot c^2 = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r_e} \quad \text{(static case)} \tag{16}$$

The kinetic energy E_k of the electron is therefore equal to:

$$E_k = \frac{m_e \cdot c^2}{\sqrt{1 - \frac{\upsilon^2}{c}}} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r_e \cdot \sqrt{1 - \frac{\upsilon^2}{c}}} \tag{17}$$

For the **<u>particular case</u>** where the speed is $\upsilon \approx 0.999973 \cdot c$, the Lorentz factor is equal to the fine structure constant. It is the situation that we think prevails at the outer limits of the luminous universe.

$$\alpha = \sqrt{1 - \frac{\upsilon^2}{c^2}} \tag{18}$$

This makes the fine structure constant appear on each side of the equation **(17)**:

$$E_k = \frac{m_e \cdot c^2}{\alpha} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r_e \cdot \alpha} \tag{19}$$

Let's suppose two electrons traveling collinearly, at the speed υ, and separated by an arbitrary distance *d*. The kinetic energy of one of these electrons would then be equal to:

$$E_k = \frac{m_e \cdot c^2 \cdot r_e}{\alpha \cdot d} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot d \cdot \alpha} \tag{20}$$

In a static case, the gravitational energy between two masses *m* located at a distance *d* from each other is given by the equation of Newton:

$$E_g = \frac{G \cdot m^2}{d} \quad \text{(static case)} \tag{21}$$

In a dynamic case where a mass *m* moves at a speed υ, we must consider the mass in movement m' with the help of the following special relativity equation:

$$m' = \frac{m}{\sqrt{1-\frac{\upsilon^2}{c^2}}} \tag{22}$$

If we consider the equation **(21)** from a dynamic point of view for the particular case of electrons with mass m_e traveling at the speed υ, the equation becomes:

$$E_g = \frac{G \cdot m_e^2}{d \cdot \left(1-\frac{\upsilon^2}{c^2}\right)} = \frac{G \cdot m_e^2}{d \cdot \alpha^2} \quad \text{(dynamic case)} \tag{23}$$

The ratio between equation **(20)** and **(23)** is the following:

$$\frac{E_k}{E_g} = \frac{\left(\frac{m_e \cdot c^2 \cdot r_e}{\alpha \cdot d}\right)}{\left(\frac{G \cdot m_e^2}{d \cdot \alpha^2}\right)} = \frac{\alpha \cdot r_e \cdot c^2}{G \cdot m_e} \tag{24}$$

According to equation **(15)**, the equation **(24)** may be rewritten in the following manner:

$$\frac{E_k}{E_g} = \frac{\alpha \cdot r_e \cdot c^2}{G \cdot m_e} = \beta \cdot N^{1/3} \tag{25}$$

The factor $\beta \cdot N^{1/3}$ therefore represents a scaling factor between the kinetic energy of an electron and its potential energy with respect to another electron that moves collinearly at a speed υ. The distance between the two electrons does not matter since we have made the calculation with a distance d that can be whatever we may wish.

Let's isolate G from the equation **(25)** to obtain:

$$G = \frac{c^2 \cdot r_e \cdot \alpha}{m_e \cdot \beta \cdot N^{1/3}} \tag{26}$$

If we use the equation **(12)** in the equation **(26)**, we obtain:

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.6732309 \pm 0.0000003 \times 10^{-11}\ \mathrm{m}^3/\left(kg \cdot s^2\right) \tag{27}$$

This value is in perfect accordance with the actual value given by the CODATA 2010 [8] which evaluates $G \approx 6.67384 \pm 0.00080 \times 10^{-11} \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$.

Without performing an analysis in the present document, the equation **(27)** lets us think that G is not constant over time, or in space.

G depends on the square of the speed of light. In previous works, we have already demonstrated that the speed of light was not constant over time. Moreover, this is the cause of the perception of a supposed «Pioneer Effect» which is in fact but an illusion. The latter is caused by the presumption that the speed of light is constant when we use the Doppler Effect. In fact, light accelerates over time and gives the impression that objects slow down.

Moreover, G depends on β. This means that the value of G is correct only for the position that we occupy here in the material universe. We cannot come to a decisive opinion about this since we do not know to what extent the classical radius of the electron r_e and its mass m_e may be affected by the position in space.

However, the value of G probably does not change very quickly since we always occupy, in proportion, the same position in space. Effectively, even if the material universe is expanding, the luminous universe is also expanding.

Let's recall that Dirac has already mentioned that he thought that all the great constants of physics were probably not really constant over time.

3. DEMONSTRATION OF THE HYPOTHESIS OF THE STARTING POINT

Previously, we used our intuition to come to the G equation. Let's try to see if we can start from a known result and get back to the equation of the starting point **(12)**.

Let's start from the following energy equation for an electron (from the wave-particle duality):

$$m_e \cdot c^2 = \frac{h \cdot c}{2 \cdot \pi \cdot r_c} \quad \textbf{(28)}$$

Here, r_c is the Compton radius of the electron. Its value may be written as a function of the classical radius of the electron r_e and the constant of the fine structure α :

$$r_c = \frac{r_e}{\alpha} \quad \textbf{(29)}$$

Then, the equation **(28)** becomes:

$$m_e \cdot c^2 = \frac{h \cdot c \cdot \alpha}{2 \cdot \pi \cdot r_e} \quad \textbf{(30)}$$

After doing a few algebraic manipulations from the equation **(30)** only, we get:

$$\frac{c^2}{\left(\sqrt{\frac{h\cdot\left(\frac{c^2\cdot r_e\cdot\alpha^{20}}{m_e}\right)}{2\cdot\pi\cdot c^3}}\cdot\left(\frac{\left(\frac{c^2\cdot r_e\cdot\alpha^{20}}{m_e}\right)\cdot m_e}{c\cdot\alpha\cdot r_e^2}\right)\right)^2}=\frac{1}{\alpha^{57}} \tag{31}$$

Let's make the β factor appear in certain strategic places without changing the result of the equation:

$$\frac{c^2}{\left(\sqrt{\frac{h\cdot\left(\frac{c^2\cdot r_e\cdot\alpha^{20}}{m_e\cdot\beta}\right)}{2\cdot\pi\cdot c^3}}\cdot\left(\frac{\left(\frac{c^2\cdot r_e\cdot\alpha^{20}}{m_e\cdot\beta}\right)\cdot m_e\cdot\beta^{3/2}}{c\cdot\alpha\cdot r_e^2}\right)\right)^2}=\frac{1}{\alpha^{57}} \tag{32}$$

Using the equation **(27)** which defines *G*, the equation **(32)** may be rewritten like this:

$$\frac{c^2}{\left(\sqrt{\frac{h\cdot G}{2\cdot\pi\cdot c^3}}\cdot\left(\frac{G\cdot m_e\cdot\beta^{3/2}}{c\cdot\alpha\cdot r_e^2}\right)\right)^2}=\frac{1}{\alpha^{57}} \tag{33}$$

Using equation **(1)**, the equation **(33)** may be rewritten like this:

$$\frac{c^2}{\left(\sqrt{\frac{h\cdot G}{2\cdot\pi\cdot c^3}}\cdot H_0\right)^2}=\frac{1}{\alpha^{57}} \tag{34}$$

It is possible to rewrite the equation like this:

$$\frac{2\cdot\pi\cdot c^3\cdot c^2}{h\cdot G\cdot H_0^2}=\frac{1}{\alpha^{57}} \tag{35}$$

This equation may be rewritten as follows:

$$\frac{\left(\frac{c^3}{G\cdot H_0}\right)}{\left(\frac{h\cdot H_0}{2\cdot\pi\cdot c^2}\right)}=\frac{1}{\alpha^{57}} \tag{36}$$

Using equations **(3)** and **(5)**, we get:

$$\frac{m_u}{m_{ph}} = \frac{1}{\alpha^{57}} \tag{37}$$

Thanks to the equation **(6)**, we deduce that:

$$N = \frac{1}{\alpha^{57}} \tag{38}$$

This is what we wanted to demonstrate.

4. REEVALUATION OF SOME PARAMETERS OF PHYSICS

If our hypotheses are correct, the accuracy of the numerical evaluation of the G constant would be greatly improved by our calculations. Since many physics parameters are related to the G constant, it would be interesting to reevaluate these more accurately. These parameters are: the apparent mass of the universe m_u, the apparent radius of curvature of the material universe r_u and luminous universe R_u, the Hubble constant H_0, the average temperature of the cosmic microwave background of the universe T, the acceleration of light a_L and the Pioneer acceleration a_p.

Let's compile the results in a tabular form.

Parameter	Equations and Values	Source
Universal gravitational constant G	$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta}$	
	$6.67323036 \pm 0.00000030 \times 10^{-11}\ \text{m}^3 / \left(kg \cdot s^2\right)$	
	$6.67384 \pm 0.00080 \times 10^{-11}\ \text{m}^3 / \left(kg \cdot s^2\right)$	CODATA 2010 [8]
Hubble constant H_0	$H_0 = \frac{G \cdot m_e \cdot \beta^{3/2}}{c \cdot \alpha \cdot r_e^2} = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e}$	
	$72.09548632 \pm 0.00000046\ \text{km} / \left(s \cdot MPar\sec\right)$	
	$72.1 \pm 0.9\ \text{km} / \left(s \cdot MPar\sec\right)$	Xiaofeng Wang [6]
Average temperature of the cosmic microwave background of the universe T	$T = \frac{1}{k_B \cdot r_e} \cdot \sqrt{\frac{m_e}{\pi^3} \cdot \sqrt{\frac{15}{8} \cdot G \cdot h^3 \cdot c^3 \cdot \beta^7}} = \frac{m_e \cdot c^2}{k_B} \cdot \left(\frac{15 \cdot \beta^6 \cdot \alpha^{17}}{\pi^3}\right)^{1/4}$	
	$2.7367951 \pm 0.0000026\ {}^\circ\text{K}$	
	$2.736 \pm 0.017\ {}^\circ\text{K}$	Cobra rocket [11]

Parameter	Equations and values	Source
Apparent mass of the universe m_u	$m_u = \frac{c^3}{G \cdot H_0} = \frac{m_e \cdot \beta^{1/2}}{\alpha^{39}}$	
	$1.728098373 \pm 0.000000079 \times 10^{53}$ kg	
	$m_u \propto \frac{c^3}{G \cdot H_0}$	Joel C. Carvalho [5]
Apparent radius of curvature of the material universe r_u	$r_u = \frac{\beta \cdot c}{H_0} = \frac{r_e \cdot \beta^{1/2}}{\alpha^{19}}$	
	$9.802071987 \pm 0.000000001 \times 10^{25}$ m	
Apparent radius of curvature of the luminous universe R_u	$R_u = \frac{c}{H_0} = \frac{r_e}{\alpha^{19} \cdot \beta^{1/2}}$	
	$1.283107880 \pm 0.00000001 \times 10^{26}$ m	
	$\frac{c}{H_0} \approx 1.28 \pm 0.016 \times 10^{26}$ m if $H_0 \approx 72.1/(s \cdot MPar\sec)$	Many sources
Acceleration of light a_L and Pioneer acceleration a_p	$a_p = -a_L = \frac{-c \cdot H_0}{\beta} = \frac{-c^2 \cdot \alpha^{19}}{r_e \cdot \beta^{1/2}}$	
	$a_p \approx -9.16903263 \pm 0.00000001 \times 10^{-10}$ m/s^2	
	$a_p \approx -8.74 \pm 1.3 \times 10^{-10}$ m/s^2	Brownstein & Moffat [13]

Several other parameters could then be recalculated (Planck time, Planck mass, Planck length, etc.). We leave it to others to re-evaluate and verify them.

5. CONCLUSION

We straightaway agree that our approach is rather closer to a conjecture. Effectively, we do not present any theory that can adequately explain the profound reason of why the inverse of the fine structure constant has to be applied 57 times to give *N*. However, we hope that our studies will enable others to identify this profound reason.

Despite everything, we think that it is not necessary to know exactly the underlying process leading to this phenomenon to doubt that it exists.

Similarly, if we see boiling water at ambient pressure, we may suspect, without necessarily knowing it well, that a source of energy is transferred to water. If we know the quantity of water, we can even calculate the minimum amount of power required. But that does not tell us whether it is an electrical power source, burning gas or other. It is somewhat the same here. We are able to see that the amounts are probably correct, but we still do not know the underlying reasons. Thus this represents the limit of our document. At the same time, it would be interesting, in a future paper, to attempt to find the underlying reasons for this phenomenon.

As we noted in our research, the "constant" G does not seem to be constant over time or space. It would be interesting in future research to find the variations that depend on these two parameters.

Our assumptions have allowed us to find an equation linking N to α directly. Knowing N indirectly with high accuracy enables us to calculate the universal gravitational constant G. Of course, since many other "constants" and parameters of the universe are defined as a function of G, it becomes possible to refine the error margins thereof. These margins of error depend on the accuracy of our hypotheses. They certainly pave the way for some mathematical explorations which may be used to describe the physical world around us.

6. REFERENCES

[1] Dirac, P. A. M., "Cosmological Models and the Large Numbers Hypothesis", *Proceedings of the Royal Society*, Great Britain, 1974, pp. 439-446.

[2] Mercier, Claude, "Large Number Hypothesis of Dirac Leading to the Hubble Constant and to the Temperature of the Cosmic Microwave Background of the Universe", *Pragtec*, Baie-Comeau, Québec, Canada, February 4, 2013, paper available on Internet at: www.pragtec.com/physique/

[3] Mercier, Claude, "The Speed of Light May Not Be Constant", *Pragtec*, Baie-Comeau, Quebec, Canada, October 8, 2011, paper available on Internet at: www.pragtec.com/physique/

[4] Mercier, Claude, "Calculation of the Apparent Mass of the Universe", *Pragtec*, Baie-Comeau, Quebec, Canada, May 5, 2012, paper available on Internet at: www.pragtec.com/physique/

[5] Carvalho, Joel C., "Derivation of the Mass of the Observable Universe", *International Journal of Theoretical Physics*, v. 34, no 12, December 1995, p. 2507.

[6] Wang, Xiaofeng and al., “Determination of the Hubble Constant, the Intrinsic Scatter of Luminosities of Type Ia SNe, and Evidence for Non-Standard Dust in Other Galaxies”, March 2011, pp. 1-40, arXiv:astro-ph/0603392v3

[7] Macleod, Alasdair, "Evidence for a Universe Expending at the Speed of Light", *University of Highlands and Islands physics*, Scotland, UK, April 2004.

[8] "Latest (2010) Values of the Constants", NIST Standard Reference Database 121, last update: April 2012, paper available on Internet at: http://physics.nist.gov/cuu/Constants/index.html

[9] Einstein, Albert, "La relativité", *Petite Bibliothèque Payot*, v. 25, Parish, original 1956 edition from Gauthier-Villar used integrally by the Payot & Rivages edition for the 2001 version, p. 109.

[10] Einstein, Albert, "On the Electrodynamics of Moving Bodies ", *The Principle of Relativity (Dover Books on Physics)*, New York, Dover publications, 1952 (original 1905 paper), pp. 35-65.

[11] Gush, H.P. and al., "Rocket Measurement of the Submillimeter Cosmic Background Spectrum", *Physical Review Letters*, v. 47, version 10, 1981, pp. 745-748.

[12] Hubble, E. and Humason, M. L., "The Velocity-Distance Relation among Extra-Galactic Nebulae", *The Astrophysical Journal*, v. 74, 1931, p.43.

[13] Brownstein , J. R. and J. W. Moffat, "Gravitational Solution to the Pioneer 10/11 Anomaly", *Classical and Quantum Gravity*, vol. 23, no. 10, 2006, pp. 3427-3436.